§5.7 Beilinson resolution

<u>Setup</u> Let $V \simeq \mathbb{C}^{n+1}$, $\mathbb{P} := \mathbb{P}(V) \simeq \mathbb{P}^n$.

Let $\varepsilon : \mathbb{P}_\Delta \hookrightarrow \mathbb{P} \times \mathbb{P}$ be the diagonal.

There's a <u>canonical</u> resolution of $\varepsilon_* \mathcal{O}_{\mathbb{P}_\Delta}$ by locally free sheaves on $\mathbb{P} \times \mathbb{P}$, called the <u>Beilinson resolution</u>:

$$0 \to \mathcal{O}_{\mathbb{P}}(-n) \boxtimes \Omega^n_{\mathbb{P}}(n) \to \mathcal{O}_{\mathbb{P}}(-n+1) \boxtimes \Omega^{n-1}_{\mathbb{P}}(n-1) \to \cdots \to \mathcal{O}_{\mathbb{P}}(-1) \boxtimes \Omega^1(1) \to \underbrace{\mathcal{O}_{\mathbb{P}} \boxtimes \mathcal{O}_{\mathbb{P}}}_{= \mathcal{O}_{\mathbb{P}\times\mathbb{P}}} \to \varepsilon_* \mathcal{O}_{\mathbb{P}_\Delta} \to 0.$$

As an immediate consequence:

<u>Cor</u> The Kunneth formula holds for $\mathbb{P}^n$.

<u>Remark</u> Also, can prove the projective bundle thm this way.