§5.6 Kunneth formula

Setup Let X be a G-variety. Write $\Delta \in K^G(X \times X)$ for the structure sheaf of the diagonal $X_\Delta \hookrightarrow X \times X$.

If X is smooth and compact, then for any G-variety Y, we have a convolution map $K^G(Y \times X) \otimes_{R(G)} K^G(X) \to K^G(Y)$.

G – linear alg. group
X – smooth projective G-variety
Y – arbitrary/any G-variety

Thm The following are equivalent.

1) $\pi : K^G(X) \otimes_{R(G)} K^G(Y) \longrightarrow K^G(X \times Y)$ is an iso.
$\mathcal{F}, \mathcal{G} \longmapsto \mathcal{F} \boxtimes \mathcal{G}$

2) Let $Y = X$. Then $\Delta \in \mathrm{im}(\pi : K^G(X) \otimes_{R(G)} K^G(X) \to K^G(X \times X))$.

3) $K^G(X)$ is a finitely generated projective $R(G)$-module.
Also, the convolution map $K^G(Y \times X) \to \mathrm{Hom}_{R(G)}(K^G(X), K^G(Y))$

is an iso.

4) - $K^G(X)$ is a f.g. proj. $R(G)$-mod.
- $K^G(X \times X)$ is a f.g. proj. $R(G)$-mod.
- $\mathrm{rk}\, K^G(X \times X) = (\mathrm{rk}\, K^G(X))^2$
- bilinear pairing $K^G(X) \times K^G(X) \to R(G)$ is non-degen. in the sense that $K^G(X) \xrightarrow{\sim} \mathrm{Hom}_{R(G)}(K^G(X), R^G(X))$.